707.000
Web Science and Web Technology
„The Small World Problem“

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Overview

Topics

• Definition of the Small World Problem
• Results from a social experiment
Do I know somebody in …?

XING

Your connection to Sa Li

Dr. Markus Strohmaier
Graz University of Technology

Dr. Harald Heiz
DFKI GmbH

Ivo Vollrath
SAP AG

Dr. Marcell Assan
MAN Nutzfahrzeuge

Sa Li
BENO mobile beijing

Sa Li
Quality gate engineer
BENO mobile beijing
Beijing, China

Options
Add as contact
Send message
Introduce
Bookmark
Show location
Show route

Menu:
Create memo

Sa Li's statistics

No Premium Membership
Member since: 10/2008
Profile hits: 228
Direct contacts: 25

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<tr>
<td>Haves</td>
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<tr>
<td>Company</td>
<td>BENO mobile beijing Quality gate engineer</td>
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(08/2006 - )
The Bacon Number

http://www.imdb.com
The Kevin Bacon Game

The oracle of Bacon

www.oracleofbacon.org
### The Bacon Number

[Watts 2002]

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The Erdös Number

Who was Erdös?
http://www.oakland.edu/enp/

A famous Hungarian Mathematician, 1913-1996
Erdös posed and solved problems in number theory and other areas and founded the field of discrete mathematics.

• 511 co-authors (Erdös number 1)
• ~ 1500 Publications
The Erdös Number

The Erdös Number:
Through how many research collaboration links is an arbitrary scientist connected to Paul Erdös?

What is a research collaboration link?
Per definition: Co-authorship on a scientific paper -> Convenient: Amenable to computational analysis

What is my Erdös Number?

⇒ 4

me -> H. Maurer -> W. Kuich -> N. Sauer -> P. Erdös
Stanley Milgram

- A social psychologist
- Yale and Harvard University

- Study on the Small World Problem, beyond well defined communities and relations (such as actors, scientists, …)

- Controversial: The Obedience Study

- What we will discuss today: „An Experimental Study of the Small World Problem”
Introduction

The simplest way of formulating the small-world problem is: **Starting with any two people in the world, what is the likelihood that they will know each other?**

A somewhat more sophisticated formulation, however, takes account of the fact that while person X and Z may not know each other directly, they may share a mutual acquaintance - that is, a person who knows both of them. One can then think of an acquaintance chain with X knowing Y and Y knowing Z. Moreover, one can imagine circumstances in which X is linked to Z not by a single link, but by a series of links, X-A-B-C-D…Y-Z. That is to say, person X knows person A who in turn knows person B, who knows C… who knows Y, who knows Z.

[Milgram 1967, according to http://www.ils.unc.edu/dpr/port/socialnetworking/theory_paper.html#2]
An Experimental Study of the Small World Problem [Travers and Milgram 1969]

A Social Network Experiment tailored towards
- Demonstrating
- Defining
- And measuring
Inter-connectedness in a large society (USA)

A test of the modern idea of “six degrees of separation”
Which states that: every person on earth is connected to any other person through a chain of acquaintances not longer than 6
Experiment

Goal
• Define a single target person and a group of starting persons
• Generate an acquaintance chain from each starter to the target

Experimental Set Up
• Each starter receives a document
• was asked to begin moving it by mail toward the target
• Information about the target: name, address, occupation, company, college, year of graduation, wife’s name and hometown
• Information about relationship (friend/acquaintance) [Granovetter 1973]

Constraints
• starter group was only allowed to send the document to people they know and
• was urged to choose the next recipient in a way as to advance the progress of the document toward the target
Questions

• How many of the starters would be able to establish contact with the target?
• How many intermediaries would be required to link starters with the target?
• What form would the distribution of chain lengths take?
Set Up

- **Target person:**
  - A Boston stockbroker
- **Three starting populations**
  - 100 “Nebraska stockholders”
  - 96 “Nebraska random”
  - 100 “Boston random”
Results I

- How many of the starters would be able to establish contact with the target?
  - 64 out of 296 reached the target

- How many intermediaries would be required to link starters with the target?
  - Well, that depends: the overall mean 5.2 links
  - Through hometown: 6.1 links
  - Through business: 4.6 links
  - Boston group faster than Nebraska groups
  - Nebraska stockholders not faster than Nebraska random

- What form would the distribution of chain lengths take?
Results II

• Incomplete chains

What reasons can you think of for incomplete chains?
Results III

- Common paths
- Also see: Gladwell’s “Law of the few”
6 degrees of separation

• So is there an upper bound of six degrees of separation in social networks?
  
  – Extremely hard to test
  – In Milgram’s study, ~2/3 of the chains didn’t reach the target
  – 1/3 random, 1/3 blue chip owners, 1/3 from Boston
  – Danger of loops (mitigated in Milgram’s study through chain records)
  – Target had a “high social status” [Kleinfeld 2000]
Follow up work (2008)

- Horvitz and Leskovec study 2008
- 30 billion conversations among 240 million people of Microsoft Messenger
- Communication graph with 180 million nodes and 1.3 billion undirected edges
- Largest social network constructed and analyzed to date (2008)
Follow up work (2008)

Approximation of “Degrees of separation”
- Random sample of 1000 nodes
- for each node the shortest paths to all other nodes was calculated. The average path length is 6.6. median at 7.
- Result: a random pair of nodes is 6.6 hops apart on the average, which is half a link longer than the length reported by Travers/Milgram.
- The 90th percentile (effective diameter (16)) of the distribution is 7.8. 48% of nodes can be reached within 6 hops and 78% within 7 hops.
- we find that there are about “7 degrees of separation” among people.
- long paths exist in the network; we found paths up to a length of 29.
Small Worlds
http://www.infosci.cornell.edu/courses/info204/2007sp/

- Every pair of nodes is connected by a path with an extremely small number of steps (low diameter)
- Nodes are clustered, e.g. people share friends
- Two principle ways of encountering small worlds
  - Dense networks
  - Sparse networks with well-placed connectors
Small Worlds
[Newman 2003]

• The small-world effect exists, if
  – “The number of vertices within a distance r of a typical central vertex grows exponentially with r (the larger it get, the faster it grows) $x(t) = x_0 e^{kt}$

In other words:
  – Networks are said to show the small-world effect if the value of l (avg. shortest distance) scales logarithmically or slower with network size for fixed mean degree $e^{\ln(x)} = x$ if $x > 0$
Formalizing the Small World Problem

[Watts and Strogatz 1998]

The small-world phenomenon is assumed to be present when

\[ L \geq L_{\text{random}} \text{ but } C > C_{\text{random}} \]

Or in other words: We are looking for networks where local clustering is high and global path lengths are small

What’s the rationale for the above formalism?

One potential answer:
Cavemen and Solaris Worlds
The Solaris World
Random Social Connections

How do random social graphs differ from „real“ social networks?

http://vimeo.com/9669721

The Cave Men World
Highly Clustered Social Connections
Reminder - previous informal definition: SMP exists when every pair of nodes in a graph is connected by a path with an extremely small number of steps. **Does not take searchability into account. Random networks are hard to search with local knowledge.**

Under which conditions can these two requirements be reconciled?

Two seemingly contradictory requirements for the Small World Phenomenon:

- It should be possible to connect two people chosen at random via chain of only a few intermediaries (as in Solaria world)
- Network should display a large clustering coefficient, so that a node's friends will know each other (as in Caveman world)
Formalizing the Small World Problem

[Watts 2003]

- Page 76 -82
- The alpha parameter
- Path length: computed only over nodes in the same connected component

Figure 3.3. Path length as a function of alpha ($\alpha$). At the critical alpha value, many small clusters join to connect the entire network, whose length then shrinks rapidly.
Demo – Small Worlds the Alpha Model

http://kmi.tugraz.at/staff/markus/demos/sw-alpha.htm

Small World Simulation - The Alpha Model
Formalizing the Small World Problem

[Watts 2003]

• Page 76 -82

• Comparison between path length and clustering coefficient

Small World Phenomenon exists when

\[ L \geq L_{\text{random}} \text{ but } C \gg C_{\text{random}} \]

Q: Why does this area not qualify to represent a small world network?

A: Not all components are connected yet (unconnected caves)
Examples for Small World Networks

[Watts and Strogatz 1998]

Table 1 Empirical examples of small-world networks

<table>
<thead>
<tr>
<th></th>
<th>$L_{\text{actual}}$</th>
<th>$L_{\text{random}}$</th>
<th>$C_{\text{actual}}$</th>
<th>$C_{\text{random}}$</th>
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<tr>
<td>Film actors</td>
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<td>2.99</td>
<td>0.79</td>
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<td>Power grid</td>
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<td>12.4</td>
<td>0.080</td>
<td>0.005</td>
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<tr>
<td>C. elegans</td>
<td>2.65</td>
<td>2.25</td>
<td>0.28</td>
<td>0.05</td>
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</table>

Characteristic path length $L$ and clustering coefficient $C$ for three real networks, compared to random graphs with the same number of vertices ($n$) and average number of edges per vertex ($k$). (Actors: $n = 225,226, k = 61$. Power grid: $n = 4,941, k = 2.67$. C. elegans: $n = 282, k = 14$.) The graphs are defined as follows. Two actors are joined by an edge if they have acted in a film together. We restrict attention to the giant connected component of this graph, which includes $\sim 90\%$ of all actors listed in the Internet Movie Database (available at http://us.imdb.com), as of April 1997. For the power grid, vertices represent generators, transformers and substations, and edges represent high-voltage transmission lines between them. For C. elegans, an edge joins two neurons if they are connected by either a synapse or a gap junction. We treat all edges as undirected and unweighted, and all vertices as identical, recognizing that these are crude approximations. All three networks show the small-world phenomenon: $L \gg L_{\text{random}}$ but $C \gg C_{\text{random}}$. 
Formalizing the Small World Problem

[Watts 2003]

- Page 87
- The beta

Figure 3.6. Construction of the beta model. The links in a one-dimensional, periodic lattice are randomly rewired with probability beta ($\beta$). When beta is zero (left), the lattice remains unchanged, and when beta is one (right), all links are rewired, generating a random network. In the middle, networks are partly ordered and partly random (for example, the original link from A to B has been rewired to $B_{\text{new}}$).
Formalizing the Small World Problem
[Watts 2003]

- Page 87 -90
- The beta parameter

Figure 3.7. Path length and clustering coefficient in the beta model.
As with the alpha model (see Figure 3.4), small-world networks exist when path length is small and the clustering coefficient is large (shaded region).
Demo – Small Worlds

http://projects.si.umich.edu/netlearn/NetLogo4/SmallWorldWS.html
Contemporary Software

- Where does the small-world phenomenon come into play in contemporary software, in organizations, ..?
  
- Xing, LinkedIn, Myspace, Facebook, FOAF, …
- Business Processes, Information and Knowledge Flow
Preferential Attachment
[Barabasi 1999]

„The rich getting richer“

Preferential Attachment refers to the high probability of a new vertex to connect to a vertex that already has a large number of connections.

Example:
1. a new website linking to more established ones
2. a new individual linking to well-known individuals in a social network
Preferential Attachment Example

Which node has the highest probability of being linked by a new node in a network that exhibits traits of preferential attachment?

Example

![Network Diagram]

FIG. 1 A small example network with eight vertices and ten edges.

[Newman 2003]
Assortative Mixing (or Homophily) [Newman 2003]

Assortative Mixing refers to selective linking of nodes to other nodes who share some common property

- E.g. degree correlation
  high degree nodes in a network associate preferentially with other high-degree nodes

- E.g. social networks
  nodes of a certain type tend to associate with the same type of nodes (e.g. by race)
Assortative Mixing (or Homophily) [Newman 2003]

FIG. 8 Friendship network of children in a US school. Friendships are determined by asking the participants, and hence are directed, since A may say that B is their friend but not vice versa. Vertices are color coded according to race, as marked, and the split from left to right in the figure is clearly primarily along lines of race. The split from top to bottom is between middle school and high school, i.e., between younger and older children. Picture courtesy of James Moody.
Disassortativity refers to selective linking of nodes to other nodes who are different in some property

- E.g. the web
  low degree nodes tend to associate with high degree nodes
Affiliation Networks

- Separate social and network structure
- Social structure implies two types of nodes, e.g. actors and movies
- Actors are connected to movies they acted in, and vice versa
- Co-Acting network is then constructed in the following way
  - a given actor is connected to all actors from a given movie
  - we repeat this procedure for all movies
Affiliation Networks

Exploring complex networks (S. Strogatz, 2001),
http://www.nature.com/nature/journal/v410/n6825/full/410268a0.html

Markus Strohmaier, Denis Helic 2010
Affiliation Networks

- Small-world networks arise naturally
- Divide nodes in two groups that reflect the social context, e.g. actors and movies, people and professions (hobbies), etc.
- But also information networks, e.g. tags and photos, hashtags and tweets in twitter, etc.
- Projection on one type of nodes, e.g. actors, people, tags, hashtags is always a small-world
- Why is this the case?
Affiliation Networks

• By definition every actor in a movie is connected to every other actor in that movie
• This is a fully connected clique of actors – local clustering is high
• Networks are then networks of overlapping cliques – locked together by actors acting in multiple movies
• By „randomly“ connecting actors to movies we obtain a network with low diameter
• High local clustering + low diameter = small-world networks
Any questions?

See you next week!